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How to measure the pomeron phase in diffractive dipion photoproduction

I.F. Ginzburg[1,a], I.P. Ivanov[1,2,b]

[1] Sobolev Institute of Mathematics, Siberian Branch of the Russian Academy of Sciences, Acad. Koptyug avenue, 4 630090 Novosibirsk, Russia
[2] Università della Calabria, Dipartimento di Fisica, INFN, Gruppo Collegato di Cosenza, Ponte P. Bucci – cubo 31C, 87036 Arcavacata di Rende (Cosenza), Italy

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Abstract. The study of charge asymmetry of pions in the high-energy process $ep \to e\pi^+\pi^- p$ ($\gamma p \to \pi^+\pi^- p$) at very small dipion momenta offers a method to measure the phase of the forward hadronic (quasi-) elastic amplitude $\gamma p \to \rho p$. We estimate the potential of such measurements at HERA.

1 Introduction

The phase δ_{F} of the forward amplitude of the hadronic elastic scattering

$$\mathcal{A} = |\mathcal{A}|\mathrm{e}^{\mathrm{i}\delta_{\mathrm{F}}} \equiv |\mathcal{A}| \exp\left[\mathrm{i}\frac{\pi}{2}(1+\Delta_{\mathrm{F}})\right] \tag{1}$$

at high energy, treated often as a pomeron phase, is an important object in hadron physics.

However, the object studied in modern experiments and dubbed the pomeron seems to be complex. In some models it is the same for all processes, in other models it is process dependent, which manifests itself in different effective intercepts in different processes. The measurement of the phase of this object in various processes will be a useful step towards clarification of its nature. For example, in the naive Regge-pole pomeron model, this phase is related directly to the pomeron intercept, $\Delta_{\mathrm{F}} = -(\alpha_{IP} - 1)$, in the model of a dipole pomeron, $\Delta_{\mathrm{F}} = -(\alpha_{IP} - 1) - \pi/(2\ln(s/s_0))$, [2], for the model with Regge pole and cuts one adds to the value given by pomeron pole intercept the contribution of the branch cut with process-dependent coefficient.

Up to this moment, the phase of such a type was measured at very high energy only for pp, $\bar{p}p$ elastic scattering (via the study of Coulomb interference near forward direction; see for the latest results [1] and for references to earlier experiments therein). Such experiments demand a detailed measurement of the cross section at extremely low transverse momentum of the recorded particle, $p_\perp \approx \sqrt{|t|} \lesssim 30\,\mathrm{MeV}$, which translates into very small scattering angles.

Here we propose to measure a similar phase for the process $\gamma p \to \rho p$ via the study of the charge asymmetry of pions in the diffractive process

$$ep \to e\pi^+\pi^- p \quad (\gamma p \to \pi^+\pi^- p). \tag{2}$$

To describe our proposal in more detail, we denote by $p_\pm$ the momenta of the $\pi^\pm$ and

$$r^\mu = p_+^\mu - p_-^\mu, \quad k^\mu = p_+^\mu + p_-^\mu, \quad M = \sqrt{k^2}. \tag{3}$$

We propose to measure the charge asymmetry of the pions in the reaction (2) in the region

$$\begin{aligned} &(20 \div 30)\,\mathrm{MeV} < k_\perp < 100\,\mathrm{MeV}, \\ &1.1 < M < 1.4\,\mathrm{GeV}. \end{aligned} \tag{4}$$

An essential part of our description is valid also at $M < 1.1\,\mathrm{GeV}$. However, we do not present here definite predictions for an experiment for this mass region due to the complex structure of the C-even amplitude of the dipion production here. At $M > 1.4\,\mathrm{GeV}$ diffractive exclusive production of pion pairs becomes a process too rare to use in the problems considered.

The main mechanism of the reaction in this domain is diffractive photoproduction of dipions in the C-odd state (the ρ meson and its "tails", including ρ') via the "physical pomeron" – the vacuum quantum number exchange in t-channel. The phase of the *amplitude of this "physical pomeron"* (1) is the main subject of our study. Besides, dipions can be produced in the C-even state via
(i) the ρ, ω Regge exchange,
(ii) the odderon exchange and
(iii) the one-photon exchange with a proton (the Primakoff effect).

The interference of the amplitudes of the C-odd and C-even dipion production provides the charge asymmetry of the observed pion distribution. The experimental study of

[a] e-mail: ginzburg@math.nsc.ru
[b] e-mail: igivanov@cs.infn.it

this charge asymmetry is a good tool for the investigation of a number of phenomena [4].

These exchanges have a very different dependence on the transverse momentum of the dipion $k_\perp$.

The Primakoff effect is strongly peaked at small transverse momenta of the dipion $k_\perp$. It can be neglected at $k_\perp > 200\,\mathrm{MeV}$ (see details in the text below). It is natural to expect that the odderon contribution, just as the ρ/ω reggeon exchanges, has a flatter $k_\perp$ dependence, similarly to other hadronic amplitudes. Besides, the contribution of the ρ/ω reggeon exchanges was estimated to be very small in the HERA energy region [5], and according to modern data the odderon contribution is low enough, so that both these contributions can be neglected at the considered low transverse momenta (4). Therefore [6] we have the following.
(i) At $k_\perp < 100\,\mathrm{MeV}$ the charge asymmetry discussed is described by interference of the pomeron contribution with the Primakoff one, and it is sensitive to the phase δ_{F}. This sensitivity offers a method to measure δ_{F} in the discussed experiments.
(ii) At $k_\perp \approx 0.3$–$1\,\mathrm{GeV}$ the Primakoff contribution is negligible, the discussed charge asymmetry is described by an interference of the pomeron and odderon contributions, and this very experiment provides an opportunity to discover the odderon [5,6].

On the proposed experimental set-up we remark the following. We suggest to observe the dipion final state without other particles in detectors (without observation of scattered proton or electron). The pions that hit the detector have transverse momenta $p_{\pm\perp} \sim M/2 \sim 500\,\mathrm{MeV}$ with emission angles $20 \div 150\,\mathrm{mrad}$, which looks not so difficult to measure. It is the sum of the transverse momenta of the two pions $k_\perp$ that is supposed to be small and measurable. So, in order for this method to be efficient, we need a reasonable resolution of the reconstruction of each pion's transverse momentum. The choice of the lower bound in $k_\perp$ (4) corresponds to the anticipated accuracy of this measurement.

Let us stress a vital feature of our suggestion. The procedure we propose *does not* demand the measurement of very small scattering angles of pions.

The quality of this set-up can be controlled via measurement of the charge-symmetric part of the cross section (CSP) in two ways. First, the observation of events with the same pion content and recording of scattered electron with $k_{\perp e} \le 30 \div 50\,\mathrm{MeV}$ has a low efficiency. However these observations will give CSP in the considered kinematical region with good enough accuracy. Second, the known results for the CSP at higher total transverse momentum (obtained with recording of electron and proton) can be used for an extrapolation in the kinematical region under interest.

In the next section we discuss the kinematics of the process and introduce the charge asymmetric variables. In Sect. 3 we present the well known amplitudes of C-odd and C-even dipion production. In Sect. 4 we study the differential cross section and find the integral charge asymmetries. In Sect. 5 we present numerical results for γp and ep collisions. Discussion and conclusions are found in Sect. 6.

2 Kinematics

In the proposed set-up without recording of electrons, the main contribution to the $ep \to e\pi^+\pi^- p$ cross section is given by a convolution of the equivalent photon spectrum with cross section of mass shell for the $\gamma p \to \pi^+\pi^- p$ subprocess. In the considered region of the $k_\perp$ accuracy of this equivalent photon (or Weiszäcker–Williams) approximation is very high (much better than $k^2_{\perp\max}/m^2_\rho$). We stress again that the procedure we propose *does not* demand the measurement of very small scattering angles of pions. Therefore we focus first on the subprocess – the dipion quasi-elastic photoproduction off the proton $\gamma p \to \pi^+\pi^- p$ considering the limitation in $k_\perp$ (4) as that for this subprocess. The convolution with equivalent photon spectrum is considered in Sect. 4.2.

Let us first consider the pocess $\gamma p \to \pi^+\pi^- p$. The energies we have in mind correspond to the HERA energy range ($\sqrt{s_{\gamma p}} \sim 100 \div 200\,\mathrm{GeV}$). The initial momenta of the photon and proton are q and P respectively, $s = (q+P)^2$, the initial photon polarization vector is $\boldsymbol{e}$. We use the kinematical variables (3) for this process as well.

We define the z-axis as the γp collision axis and label the vectors orthogonal to this axis by $\perp$. Let us denote by z_+ and z_- the standard light cone variables for each charged pion, $z_\pm \approx (\epsilon_\pm + p_{\pm z})/(2E_\gamma) = (p_\pm P)/(qP)$ (for the considered process $z_+ + z_- = 1$).

We direct the x-axis along the vector $\boldsymbol{k}_\perp$ and define by ψ the azimuthal angle of the linear photon polarization with respect to the fixed laboratory frame of reference. For instance, for the photon in electroproduction $ep \to e\pi^+\pi^- p$, virtual photons are polarized in the electron scattering plane and ψ is the azimuthal angle relative to the electron scattering plane. Then the polarization vector of the initial photon with helicity $\lambda_\gamma = \pm 1$ can be written as $\boldsymbol{e}^\lambda = -\frac{1}{\sqrt{2}} \cdot \mathrm{e}^{-\mathrm{i}\lambda_\gamma \psi}(\lambda_\gamma, i)$.

It is useful also to consider polar and azimuthal angles of π^+ in the dipion CMS, θ and ϕ, and the velocity of a pion in this frame $\beta = \sqrt{1 - 4m^2_\pi/M^2}$, so that $r_{\mathrm{CMS}} = \beta M(0, \sin\theta\cos\phi,\ \sin\theta\cos\phi,\ cos\theta)$. We denote by J the total angular momentum (total spin) of the dipion, by λ_γ and $\lambda_{\pi^+\pi^-}$ the helicities of photon and produced dipion, respectively, and by $n = |\lambda_\gamma - \lambda_\rho|$ the value of the helicity flip for each amplitude. (The final results are averaged over initial photon polarizations.) Instead of phase analysis in terms of these angular variables, many physical problems can be solved definitely via the measurement of charge asymmetry of pions.

The phenomenon of charge asymmetry is the difference in the distributions of particles and antiparticles. It is determined by the part of the differential cross section that changes sign under a $r^\mu \to -r^\mu$ change. Particularly, we describe the forward–backward (FB) and transverse (T) asymmetries by the variables

$$\begin{aligned}
\mathrm{FB}: \quad \xi &= \frac{z_+ - z_-}{\beta(z_+ + z_-)} = \cos\theta, \\
\mathrm{T}: \quad v &= \frac{p_{+\perp}^2 - p_{-\perp}^2 - \xi k_\perp^2}{\beta M|k_\perp|} \\
&\equiv \frac{(\rho_\perp k_\perp)}{\beta M|k_\perp|} \\
&= \sin\theta\cos\phi; \quad \rho_\perp = r_\perp - \xi k_\perp. \qquad (5)
\end{aligned}$$

We consider the amplitude of the dipion production $\mathcal{A}$, which is normalized so that

$$\begin{aligned}
\mathrm{d}\sigma &= |\mathcal{A}|^2\ \beta \mathrm{d}M^2\,\mathrm{d}k_\perp^2\,\mathrm{d}\cos\theta\,\mathrm{d}\phi\,\frac{\mathrm{d}\psi}{2\pi} \\
&= \frac{2}{\sqrt{1-\xi^2-v^2}}\ |\mathcal{A}|^2\ \beta\mathrm{d}M^2\,\mathrm{d}k_\perp^2\,\mathrm{d}\xi\,\mathrm{d}v\,\frac{\mathrm{d}\psi}{2\pi}. \qquad (6)
\end{aligned}$$

We will see below that only a transverse asymmetry arises in the case considered. We describe the values of this charge asymmetry and the charge-symmetric background by the quantities

$$\Delta\sigma_T = \int \mathrm{d}\sigma(v>0) - \int \mathrm{d}\sigma(v<0), \quad \sigma_{\mathrm{bkgd}} = \int \mathrm{d}\sigma, \quad (7)$$

with integration over the (identical) suitable region of the final phase space.

3 The amplitudes

Note first that in the considered range of momentum transfer (4) the inelastic transitions in the proton vertex as well as the helicity-flip elastic transitions are small.

We now consider the C-odd dipion diffractive production. It is described by the "physical pomeron". It has been studied both in theory and in experiment (e.g. at HERA) as a production of C-odd resonances, mainly the $\rho(770)$ meson with well known properties.

Our basic assumption is that the amplitude can be written in the form

$$\mathcal{A} = \sum_{Jn} A_{Jn}(s,t,M^2) D_J(M^2) \mathcal{E}_J^{\lambda_\gamma,\lambda_{\pi\pi}}. \qquad (8)$$

(i) The first factor $A_{Jn}(s,t,M^2)$ is "the pomeron amplitude" for the production of the dipion state with effective mass M, angular momentum J and helicity flip n. In the considered mass region the contribution with $J=1$ dominates (the admixture of $J=3$ looks negligible). In our discussion we assume that in the considered mass interval the entire dependence of the amplitude $\mathcal{A}$ on the dipion mass M at $t \approx 0$ can be accumulated with a high precision in the factor $D_J(M^2)$ so that the amplitude A_{Jn} is only weakly dependent on the dipion mass M. It is normalized in the ρ-meson peak in such a manner that $A_{1n=0}(M=M_\rho) = \mathrm{e}^{\mathrm{i}\delta_{\mathrm{F}}}|A_{1n}|$, with

$$\begin{aligned}
&|A_{1n}|^2 = \sigma_\rho B \mathrm{e}^{-Bk_\perp^2} \quad \text{with} \\
&B \approx 10\,\mathrm{GeV}^{-2}, \quad \sigma_\rho \approx 11\,\mu\mathrm{b}.
\end{aligned} \qquad (9)$$

The s-channel helicity conservation $\lambda_{\pi^+\pi^-} = \lambda_\gamma$ for process (2) is a well established fact. For the considered $k_\perp$ region, the helicity violating amplitudes are as small as $\sim (|k_\perp|/M)^n \le (0.03)^n$, and we neglect them below. Lastly, in the considered region (4) the t-dependence of the amplitude is negligible.

(ii) The second factor $D_J(M)$ describes the decay of this dipion state to pions – it is driven by the strong interaction of the pions in the final state. In similarity to the construction of the pion formfactor, it should be constructed from contributions of ρ, ρ', ρ'' in a manner to describe the data in the effective mass interval considered. At $2m_\pi < M < M_\rho$ one can use for D_1 the well known Gounaris–Sakurai approximation obtained for the pion formfactor. At $M > M_\rho$ one should take into account also the ρ' etc. states with variable parameters given by the coupling constants and parameters of ρ', ρ''. A reasonable parameterization should give a complete description of the dipion mass spectrum $\propto |D_1|^2$ in the considered region[1]. Below, we use the fit from [7]. It covers the required mass interval and includes ρ running width and ρ'/ρ'' states. Note that the parameters of the model can be fixed with a better accuracy with a detailed measurement of the dominant C-even dipion mass spectrum in the very experiment we propose.

The qualitative discussion becomes transparent with the standard Breit–Wigner factor for $R=\rho(770)$ (including $R\pi^+\pi^-$ coupling)

$$D_J(M^2) \approx \frac{\sqrt{m_R \Gamma_R \mathrm{Br}(R\to\pi^+\pi^-)/\pi}}{-M^2 + m_R^2 - \mathrm{i}m_R\Gamma_R}. \qquad (10)$$

(iii) The third factor $\mathcal{E}_J^{\lambda_\gamma,\lambda_{\pi\pi}}$ describes the angular distribution of the pions in their rest frame, $\mathcal{E}_J^{\lambda_\gamma,\lambda_{\pi\pi}} = Y^{J,\lambda_{\pi\pi}}(\theta,\phi)\mathrm{e}^{-\mathrm{i}\lambda_\gamma\psi}$.

Finally, the amplitude of the C-odd dipion photoproduction reads

$$\begin{aligned}
\mathcal{A}_- &= \mathrm{e}^{\mathrm{i}\delta_{\mathrm{F}}} \cdot |A_{1,0}(s)| \cdot D_1(M^2) \cdot \sqrt{\frac{3}{8\pi}}\sin\theta \mathrm{e}^{\mathrm{i}\lambda_\gamma(\phi-\psi)} \quad (11) \\
&\equiv \mathrm{e}^{\mathrm{i}\delta_{\mathrm{F}}} \cdot |A_{1,0}(s)| \cdot D_1(M^2) \cdot \sqrt{\frac{3}{8\pi}}\sqrt{1-\xi^2}\,\mathrm{e}^{\mathrm{i}\lambda_\gamma(\phi-\psi)}.
\end{aligned}$$

Here and below the subscript, $-$ or $+$, on $\mathcal{A}$ denotes the C-parity of the produced dipion.

The amplitude of the production of C-even dipions via the Primakoff effect is the same as that in the two-photon processes $\mathrm{e}^+e^- \to \mathrm{e}^+e^-\pi^+\pi^-$ [10, 11]. In the regions of interest (4) the dominant contribution is given by the almost real photon exchanges with both electron and proton. Therefore, the total helicity of the initial two-photon state

[1] To take into account a possible dependence $B(M)$, one should distinguish here a quantity defined at $t=0$ and that obtained by an extrapolation procedure, the second quantity can contain also a factor appearing due to t-integration of $\mathrm{e}^{-B_M k_\perp^2}$. These quantities will be obtained from two different methods of verification of the proposed set-up (see the end of Sect. 1). The first method gives the quantity at $t \approx 0$, the second method *can* include integration over t.

and respectively of dipions can be 0 and 2. The amplitude can be written in a form similar to (8).

Beginning from the threshold, the pions interact strongly in the $I = J = 0$ state (which is described by f_0 resonances). The other partial waves are described well with the QED approximation for point-like pions (with known small modifications). The QED amplitude with $I = 0$, $J = 2$ become surprisingly large starting from $M = 0.5$–0.7 GeV. The other amplitudes can be neglected everywhere in our problem.

At $M^2 \ll s_{\gamma p}$ the amplitude of the Primakoff $\gamma p \to Rp$ process can be written [10] via the two-photon decay width $\Gamma^R_{\gamma\gamma}$ of the resonance R with spin J:

$$A_\gamma = \sqrt{\sigma_2} \cdot \frac{|k_\perp|}{k_\perp^2 + Q^2_{\mathrm{m}}},$$

with

$$Q^2_{\mathrm{m}} = \left(\frac{m_p M^2}{s}\right)^2, \quad \sigma_2 \equiv \frac{8\pi\alpha\Gamma^R_{\gamma\gamma}(2J+1)}{m_R^3}. \tag{12a}$$

Here, Q^2_{m} is the minimal value of the virtuality of the exchanged photon (typically $Q^2_{\mathrm{m}} < m_e^2$).

At $1.1 < M < 1.4$ GeV, the main contribution is given by the $I = 0$, $J = 2$ partial wave, the other partial waves being negligible. Here $f_2(1270)$-meson ($J = 2$) production dominates. We define by g_0 and g_2 the relative probability amplitudes of the dipion production in the states with helicity 0 and 2; $g_0^2 + g_2^2 = 1$. According to the data, the contribution of the total helicity $\lambda_{\pi\pi} = 2$ dominates, i.e. $g_2 \gg |g_0|$ (see e.g. [9]). Similarly to (11), the amplitude of the process can be written as

$$\begin{aligned}
&\mathcal{A}_+ \\
&= A_\gamma \cdot D_2(M^2) \cdot (g_2 Y_{2,2}(\theta,\phi) + g_0 Y_{2,0}(\theta,\phi))\, \mathrm{e}^{-\mathrm{i}\lambda_\gamma\psi} \\
&\equiv A_\gamma \cdot D_2(M^2) \qquad\qquad\qquad\qquad\qquad\qquad (12\mathrm{b}) \\
&\times\sqrt{\frac{15}{32\pi}}\left[g_2(1-\xi^2)\mathrm{e}^{2\mathrm{i}\lambda_\gamma\phi} + g_0\sqrt{\frac{2}{3}}(3\xi^2-1)\right]\mathrm{e}^{-\mathrm{i}\lambda_\gamma\psi},
\end{aligned}$$

with the D_2 factor given by (10) with $R = f_2(1270)$.

For a more precise calculation, the QED contribution should be also accounted for. In the unitarized model describing the data near the f_2 peak constructed in [8] the $I = 0$, $J = 2$ partial wave contains a phase-shifted Breit–Wigner factor and a modified Born QED term:

$$\begin{aligned}
D_2(M^2) = &\frac{\sqrt{m_f\Gamma_f(M^2)\mathrm{Br}(f_2 \to \pi^+\pi^-)/\pi}}{-M^2 + m_f^2 - \mathrm{i}m_f\Gamma_f(M^2)} \cdot \mathrm{e}^{\mathrm{i}\zeta} \\
&+ D_2^{\mathrm{QED}}(M^2).
\end{aligned} \tag{13}$$

The mass dependence of the f_2 width $\Gamma_f(M^2)$ and the parametrization for the modified QED contribution $D_2^{\mathrm{QED}}(M^2)$ were taken from [8]. The phase factor $\mathrm{e}^{\mathrm{i}\zeta}$ represents one particular possibility to effectively fulfill the unitarity constraint: the value of ζ is such that D_2 becomes purely imaginary at $M = m_f$.

4 Cross sections

4.1 Differential cross section. Photoproduction

The differential cross section of the $\gamma p \to \pi^+\pi^- p$ subprocess at $1.1 < M < 1.4$ GeV averaged over the initial photon polarizations is

$$\begin{aligned}
&\mathrm{d}\sigma = 2\frac{|\mathcal{A}_- + \mathcal{A}_+|^2}{\sqrt{1-\xi^2-v^2}}\beta \mathrm{d}M^2\mathrm{d}k_\perp^2\mathrm{d}\xi\mathrm{d}v \qquad\qquad (14)\\
&= \mathrm{d}\sigma_{\mathrm{sym}} + \mathrm{d}\sigma_{\mathrm{asym}}, \\
&\frac{\mathrm{d}\sigma_{\mathrm{sym}}}{\mathrm{d}M^2\mathrm{d}k_\perp^2\mathrm{d}\xi\mathrm{d}v} \\
&= \frac{2\beta}{\sqrt{1-\xi^2-v^2}}\left\{|A_{1,0}(s)|^2|D_1(M^2)|^2\frac{3}{8\pi}(1-\xi^2)\right. \\
&\quad + \frac{15}{16\pi}A_\gamma^2|D_2|^2\left[\frac{g_2^2}{2}(1-\xi^2)^2 + 3g_0^2\left(\xi^2-\frac{1}{3}\right)^2\right. \\
&\quad \left.\left. + g_0 g_2\sqrt{6}\left(\xi^2-\frac{1}{3}\right)(2v^2+\xi^2-1)\right]\right\}, \\
&\frac{\mathrm{d}\sigma_{\mathrm{asym}}}{\mathrm{d}M^2\mathrm{d}k_\perp^2\mathrm{d}\xi\mathrm{d}v} \\
&= v\cdot\frac{\beta}{\sqrt{1-\xi^2-v^2}}\cdot\frac{3\sqrt{5}}{4\pi}|A_{1,0}(s)|\,A_\gamma\cdot\mathrm{Re}\left[D_1\mathrm{e}^{\mathrm{i}\delta_{\mathrm{F}}}D_2^\dagger\right] \\
&\quad \times\left[g_2(1-\xi^2) + g_0\sqrt{\frac{2}{3}}(3\xi^2-1)\right]. \qquad\qquad (15)
\end{aligned}$$

Here $\mathrm{d}\sigma_{\mathrm{sym}}$ represents the charge-symmetric contribution, which comes from the squares of the pomeron and of the Primakoff amplitudes. The interference between these amplitudes produces the charge-asymmetric contribution $\mathrm{d}\sigma_{\mathrm{asym}}$. Since the phase δ_{F} enters only $\mathrm{d}\sigma_{\mathrm{asym}}$, we need to extract charge asymmetry; for this task the charge-symmetric contribution $\mathrm{d}\sigma_{\mathrm{sym}}$ is a background.

The appearance of the factor v, *describing transverse charge asymmetry*, in the interference term is very natural. First, due to integration over ψ, we are left with terms diagonal in the photon polarization states, i.e. λ_γ is the same in $\mathcal{A}_\pm$ and in $\mathcal{A}_\pm^\dagger$. Then, $\mathcal{A}_-\mathcal{A}_+^*$ can be rewritten as a charge-symmetric factor multiplied by $\sin\theta\mathrm{e}^{\pm\mathrm{i}\lambda_\gamma\phi}$. The averaging over the initial photon polarizations means that we sum contributions with opposite helicities, i.e. consider the sum which is proportional to $\sin\theta\mathrm{e}^{\pm\mathrm{i}\lambda_\gamma\phi} + \mathrm{h.c.} \Rightarrow v$. In other words, the averaging over photon polarization transforms complex factors from the spherical harmonics to the real factor describing charge asymmetry.

For our case when one can consider only one partial wave in the Primakoff amplitude, the M-dependence in $\mathrm{d}\sigma_{\mathrm{asym}}$ is described completely by the *overlap function*, which is independent on the g_0 and g_2 interrelation,

$$\mathcal{I}_{\rho f}(M^2) = \mathrm{Re}\left[D_1\mathrm{e}^{\mathrm{i}\delta_{\mathrm{F}}}D_2^\dagger\right]. \tag{16}$$

Let us consider the shape of the M-dependence. Figure 1 demonstrates the overlap functions for several pomeron

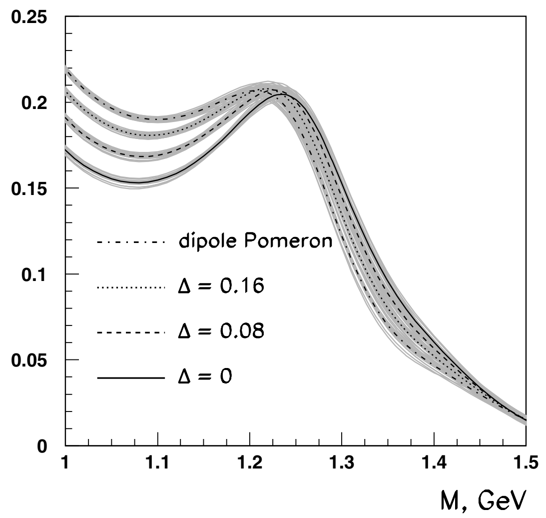

Fig. 1. Overlap function $\mathcal{I}_{\rho f}$ versus M^2. The black solid, dashed, and dotted curves correspond to a simple pole pomeron model with $\Delta_{\rm F} = 0$, 0.08, and 0.16, respectively. The dashed-dotted curve represents one particular parametrization of a dipole pomeron model, [2], calculated for $\sqrt{s_{\gamma p}} = 50\,\text{GeV}$. In each of the four cases, the grey region corresponds to 1σ variations in the parameters of the ρ', ρ'' and f_2 resonances

models. The parameterization for D_1 was taken from [7] (with the $\rho/\rho'/\rho''$ parameters and running of the ρ width taken into account), while the parametrization for D_2, which included both the f_2 resonance and the $I = 0$, $J = 2$ modified Born term, was taken from [8]. The four black curves correspond here to different pomeron models. The solid, dashed, and dotted curves correspond to the simple pole pomeron model with $\Delta_{\rm F} = 0$, 0.08, and 0.16, respectively. The dashed-dotted curve represents one particular parametrization of a dipole pomeron model [2], calculated for $\sqrt{s_{\gamma p}} = 50\,\text{GeV}$. In each of the four cases, the grey region corresponds to 1σ variations of the parameters in D_i used. The resulting shaded region allows one to see the typical level of inaccuracy that arises from the parameterizations used. It is not large, and allows one to discern different pomeron models for the $M_{\pi\pi}$ interval below the f_2 peak.

Note that many qualitative features can be easily understood in the simplified ρ-meson model for D_1, (10). It was found numerically that this approximation gives also a reasonable quantitative approximation for the overlap function.

On the dependence on the momentum transfer, note that integrating the differential cross section (7) over the whole ξ, v space, within the mentioned M interval, and with the $k_\perp$ interval $k_{\max} > k_\perp > k_{\min}$, (4), one obtains

$$\sigma_{\rm bkgd} = \sigma_\rho B C_1 (k_{\max}^2 - k_{\min}^2) + \sigma_2 C_2 \ln \frac{k_{\max}^2}{k_{\min}^2 + Q_{\rm m}^2},$$
$$C_i = \int {\rm d}M^2 |D_i(M^2)|^2, \qquad (17)$$

Numerical estimates show that the second term here, which represents the Primakoff contribution, can be neglected at the considered values of $k_{\max}$ and $k_{\min}$, (4). The integral value of the charge asymmetry, $\Delta\sigma_{\rm chas,T}$, calculated in the same M and $k_\perp$ regions, is

$$\Delta\sigma_{\rm chas,T} = \frac{9\sqrt{5}}{8}\sqrt{\sigma_\rho B \cdot \sigma_2} \cdot \Delta\mathcal{I}_{\rho f} \cdot (k_{\max} - k_{\min}),$$
$$\Delta\mathcal{I}_{\rho f} = \int {\rm d}M^2 \mathcal{I}_{\rho f}(M^2). \qquad (18)$$

To obtain simple estimates, we set $g_2 = 1$, $g_0 = 0$.

4.2 ep collisions

We think that the most efficient way to study the problem of interest is to investigate dipion production in $ep \to e\pi^+\pi^- p$, e.g. at HERA without recording of a scattered electron and proton (and without other particles in the detector except π^+ and π^-).

The ep cross section is given by a convolution of the virtual photon flux originating from the electron with the cross section of the γp subprocess. The dominant part of the ep cross section comes from the region of very small virtuality of the emitted photon. That is the base of the equivalent photon approximation (see e.g. [10]), in which the flux of the equivalent photons with energy $\omega = yE_e$ and transverse momentum $q_\perp$ is

$${\rm d}n_\gamma = \frac{\alpha}{\pi}\frac{{\rm d}y}{y}\left[1 - y + \frac{y^2}{2} - (1-y)\frac{q_e^2}{q_\perp^2}\right]\frac{q_\perp^2\,{\rm d}q_\perp^2}{(q_\perp^2 + q_e^2)^2},$$

with

$$q_e^2 = \frac{m_e^2 y^2}{1-y}. \qquad (19)$$

Note that the photon energy ω coincides with the total dipion energy with high accuracy.

The main contribution to the ep cross section originates from the region of virtualities $q_\perp^2/(1-y) + q_e^2$ much lower than the characteristic scale of hadronic interactions. Therefore,

(i) the distribution is peaked at very small $q_\perp$ when the scattered electron escapes observation;

(ii) with high enough accuracy one can take for the amplitudes of γp subprocess their on-shell values discussed above;

(iii) the precision of (19) is very high (much better than $k_{\perp,\max}^2/m_\rho^2$). At this transition from photons to electrons, the quantities (5) that describe the charge asymmetry are transformed as follows: the FB variable ξ stays unchanged since it is independent of a small change of transverse momentum and keeps its form under the longitudinal boost; the transverse variable v will be now defined by the same expression (5), but in terms of the transverse dipion momentum $\boldsymbol{K}_\perp = \boldsymbol{q}_\perp + \boldsymbol{k}_\perp$ with respect to the ep collision axis.

The differential cross section of dipion production in ep collisions is given by a convolution of the flux (19) with (14) and (15). For numerical estimates in our kinematical region (1) it is useful to change the $k_\perp^2$-dependence from (9) to $1/(1 + Bk_\perp^2)$ which is a good approximation at the considered $Bk_\perp^2 < 0.1$. In the region (4) we have also

$K_\perp^2 \gg Q_{\rm m}^2$, q_e^2 and the charge-symmetric part of cross section can be written as (see e.g. [10])

$$\frac{\mathrm{d}\sigma^{ep}_{\rm pom}}{\mathrm{d}M^2\,\mathrm{d}K_\perp^2\,\mathrm{d}\xi\,\mathrm{d}v\,\mathrm{d}y} = \frac{\beta}{\sqrt{1-\xi^2-v^2}}|A_{1,0}(s)|^2|D_1(M^2)|^2\frac{3}{4\pi}(1-\xi^2) \times\frac{\alpha}{\pi y}\left[\left(1-y+\frac{y^2}{2}\right)\cdot\left(\log\frac{1}{Bq_e^2}-1\right)-(1-y)\right],$$

$$\frac{\mathrm{d}\sigma^{ep}_{\rm Prim}}{\mathrm{d}M^2\,\mathrm{d}K_\perp^2\,\mathrm{d}\xi\,\mathrm{d}v\,\mathrm{d}y} = \frac{\beta}{\sqrt{1-\xi^2-v^2}}\sigma_2|D_2(M^2)|^2 \times\frac{15}{8\pi}\left[\frac{g_2^2}{2}(1-\xi^2)^2+3g_0^2\left(\xi^2-\frac{1}{3}\right)^2 + g_0g_2\sqrt{6}\left(\xi^2-\frac{1}{3}\right)(2v^2+\xi^2-1)\right] \times\frac{\alpha}{\pi y(K_\perp^2+Q_{\rm m}^2+q_e^2)} \times\left[\left(1-y+\frac{y^2}{2}\right)\cdot\left(\log\left(\frac{(K_\perp^2)^2}{Q_{\rm m}^2q_e^2}\right)-2\right)-(1-y)\right].$$

Note that in the Primakoff contribution we also keep the term $Q_{\rm m}^2+q_e^2$ in the denominator, which is negligible in the considered kinematical range but useful in the estimate of total cross section.

The charge asymmetric contribution is written now via the new value v as

$$\frac{\mathrm{d}\sigma^{ep}_{\rm asym}}{\mathrm{d}M^2\,\mathrm{d}K_\perp^2\,\mathrm{d}\xi\,\mathrm{d}v\,\mathrm{d}y} = \frac{v\cdot\beta}{\sqrt{1-\xi^2-v^2}}\cdot\frac{3\sqrt{5}}{4\pi}|A_{1,0}(s)|\sqrt{\sigma_2} \times\left[g_2(1-\xi^2)+g_0\sqrt{\frac{2}{3}}(3\xi^2-1)\right] \times\frac{\alpha|K_\perp|}{\pi yK_\perp^2}\left[\left(1-y+\frac{y^2}{2}\right)\cdot\left(\log\frac{K_\perp^2}{q_e^2}-\frac{1}{2}\right)-\frac{1-y}{2}\right] \times\mathcal{I}_{\rho f}(M^2). \quad (20)$$

The total values of the signal and background integrated over the entire region (4) similar to those written in (17) and (18) and with the same notation, written with logarithmic accuracy (for estimates), are

$$\frac{\sigma^{ep}_{\rm bkgd}}{\mathrm{d}y} = N_\gamma(y)\left[\sigma_\rho B\,C_1\ln\frac{1}{Bq_e^2}(K_{\max}^2-K_{\min}^2) + \sigma_2\,C_2\ln\frac{K_{\max}^2}{K_{\min}^2}\ln\frac{K_{\max}^2K_{\min}^2}{Q_{\rm m}^2q_e^2}\right], \quad (21)$$

$$\frac{\Delta\sigma^{ep}_{\rm chas,T}}{\mathrm{d}y} = N_\gamma(y)\frac{9\sqrt{5}}{8}\sqrt{\sigma_\rho B\cdot\sigma_2}\cdot\Delta\mathcal{I}_{\rho f} \times\left(K_{\max}\ln\frac{K_{\max}^2}{q_e^2}-K_{\min}\ln\frac{K_{\min}^2}{q_e^2}\right).$$

Here $N_\gamma(y) = (\alpha/\pi y)(1-y+y^2/2)$.

5 Estimates of the effect

5.1 Extracting charge asymmetry

For the integrated luminosity $\mathcal{L}$, the statistical significance of the result is given by the ratio of the number of events of interest $\mathcal{L}\Delta\sigma_{\rm chas,T}$ to the dispersion of background events $\sqrt{\mathcal{L}\,\sigma_{\rm bkgd}}$,

$$\mathrm{SS} = \frac{\mathcal{L}\Delta\sigma_{\rm chas,T}}{\sqrt{\mathcal{L}\,\sigma_{\rm bkgd}}}. \quad (22)$$

In particular, we consider the *local statistical significance* SS(M) defined by this very equation for a fixed value of the dipion mass M, $\mathrm{SS}(M) = \mathcal{L}\mathrm{d}\Delta\sigma_{\rm chas,T}/\mathrm{d}M^2/\sqrt{\mathcal{L}\,\mathrm{d}\sigma_{\rm bkgd}/\mathrm{d}M^2}$. The study of the shape of this SS(M) helps us in the choice of cuts in M for data processing.

Figure 2 shows this local statistical significance. For the C-odd dipions, as said above, we assume the ρ-meson dominance with $|A_{1,0}|$ given by (9). For f_2-meson production, σ_2 in (12a) is $\sigma_2 = 0.42\,\mathrm{nb}$. Besides, we set $g_0 = 0$, $g_2 = 1$ in accordance with the data for f_2 production in photon collisions. All other parameters were already discussed.

Let us remind the reader that the diffractive dipion photoproduction dominates over the Primakoff contribution in the C-even part of the cross section. Therefore, in the mass region, where f_2 production dominates in the Primakoff effect, the local statistical significance is estimated as

$$\mathrm{SS}(M) \propto \mathrm{Re}(D_2^*\mathrm{e}^{\mathrm{i}\delta_{\rm F}}D_1)/|D_1| \le |D_2(M)|.$$

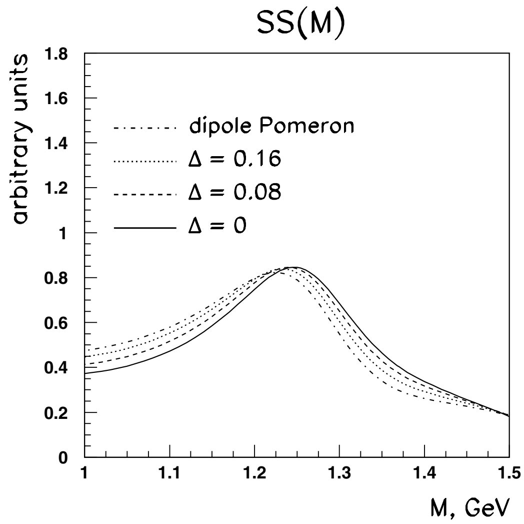

Fig. 2. The local statistical significance of the charge asymmetry (arbitrary units). The solid and dashed curves correspond to $\Delta_{\rm F} = 0$ and 0.16, respectively

This suggests that the largest statistical significance comes approximately from the region under the f_2-meson peak $m_f - \Gamma_f < M < m_f + \Gamma_f$. This is the reason why we study here the charge asymmetry only in the region $1.1 < M < 1.4\,\mathrm{GeV}$.

Integration of $|D_i|^2$ and of the overlap function $\mathcal{I}_{\rho f}$ over this range gives for the quantities in (21) at $\Delta_{\mathrm{F}} = 0$

$$\begin{aligned} \Delta\mathcal{I}_{\rho f} &= \int \mathrm{d}M^2 \mathcal{I}(M^2) = 0.114; \\ C_1 &= \int \mathrm{d}M^2 |D_1(M^2)|^2 = 0.045; \\ C_2 &= \int \mathrm{d}M^2 |D_2(M^2)|^2 = 0.36. \end{aligned} \tag{23}$$

Note that the value of $\Delta\mathcal{I}_{\rho f}$ depends on Δ_{F} only weakly.

5.2 Numerical estimates

Let us treat γp collisions. Now the statistical significance of the observation of the charge asymmetry (22) is evidently independent of the upper cut $k_{\max}$. The upper cut $k_{\max} = 100\,\mathrm{MeV}$ guarantees that the odderon contribution is negligible. The resulting cross sections are

$$\begin{aligned} &\sigma_{\mathrm{bkgd}} = 49\,\mathrm{nb}; \quad \Delta\sigma_{\mathrm{chas,T}} = 5.2\,\mathrm{nb}; \\ &\mathrm{SS}_{\mathrm{T}} \approx 24 \cdot \sqrt{\mathcal{L}_{\gamma p}(\,\mathrm{pb}^{-1})}. \end{aligned} \tag{24}$$

On the ep collisions we note the following. For the ep collisions, we take, for definiteness, $\mathcal{L}_{ep} = 100\,\mathrm{pb}^{-1}$ and integrate over the y interval $0.2 < y < 0.8$. We then obtain the following values of the cross sections and of the statistical significance:

$$\begin{aligned} &\sigma^{ep}_{\mathrm{bkgd}} \approx 1.5\,\mathrm{nb}, \\ &\Delta\sigma_{\mathrm{chas,T}} \approx 0.13\,\mathrm{nb} \quad \Rightarrow \quad \mathrm{SS}_{\mathrm{T}} \approx 34. \end{aligned} \tag{25}$$

Now for the sensitivity to δ_{F}. The above values of the integral SS show that the effect is observable at HERA with good confidence. We hope that after dedicated specification of the models for D_i, a detailed study of the M-shape of this charge asymmetry will allow for extraction of the pomeron phase δ_{F} with reasonable precision.

6 Discussion and conclusions

We showed that the interference between the pomeron exchange and the Primakoff effect contributions gives charge asymmetry in the pion distributions. The absolute value and the shape of the M-dependence of this charge asymmetry is sensitive to the phase of the strong amplitude (the pomeron phase) δ_{F}. An accurate study of this shape can lead to a direct measurement of δ_{F}. Our estimates show that this effect can be studied at HERA.

The approach suggested avoids the problems associated with the measurement of very small transverse momenta of the detected particles, in contrast to the strong–Coulomb interference in elastic pp scattering (where one should measure transverse momenta $p_\perp \lesssim 100\,\mathrm{MeV}$). Here, the detected pions have typical transverse momenta $|p_{\pm\perp}| \sim 500\,\mathrm{MeV}$ (for higher M), which looks measurable better than the proton momenta in the mentioned case of Coulomb interference.

Equations written in the text allow one to obtain a preliminary estimate for δ_{F} and find its s-dependence with an accuracy limited by the details of the experimentation. A more precise extraction of the absolute value of δ_{F} demands more accurate models both for the pomeron and Primakoff amplitudes. The main features of these models are well known, and these models can be further improved right in the course of dedicated experiments on charge asymmetries (both at high-energy lepton–hadron or low-energy e^+e^- colliders). A sketch of how predictions can be made more precise is given in the text. The invariant mass interval $M = 1.1 \div 1.3\,\mathrm{GeV}$ seems to be particularly suitable, since theoretical predictions can be made more precise here. For each mass interval, these problems should be studied separately.

Let us take the case $M < 1\,\mathrm{GeV}$. At lower dipion invariant masses, $M \lesssim 1\,\mathrm{GeV}$, the study of the transverse charge asymmetry can also be used for extraction of the pomeron phase. A more detailed model for the $\gamma\gamma \to \pi^+\pi^-$ reaction is necessary to make more accurate predictions for the study of the pomeron phase. This model can be verified by the measurement of a similar charge asymmetry in the process $e^+e^- \to e^+e^-\pi^+\pi^-$ at modern e^+e^- colliders [11]. That is the subject of forthcoming studies.

Preliminary estimates show that below the ρ peak the phases of factors D_1 and D_0 are close to each other, so that the contribution of their interference term to the considered symmetry is small. The dominant contribution to the charge asymmetry is given here by the ρ/QED interference. The best statistical significance of charge asymmetry comes from the region $M = 0.4$–$0.8\,\mathrm{GeV}$.

The extension of this idea to nuclear targets is straightforward. A detailed treatment of charge asymmetry in dipion production in eA collisions (e-RHIC or nuclear LHC) will be given elsewhere.

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